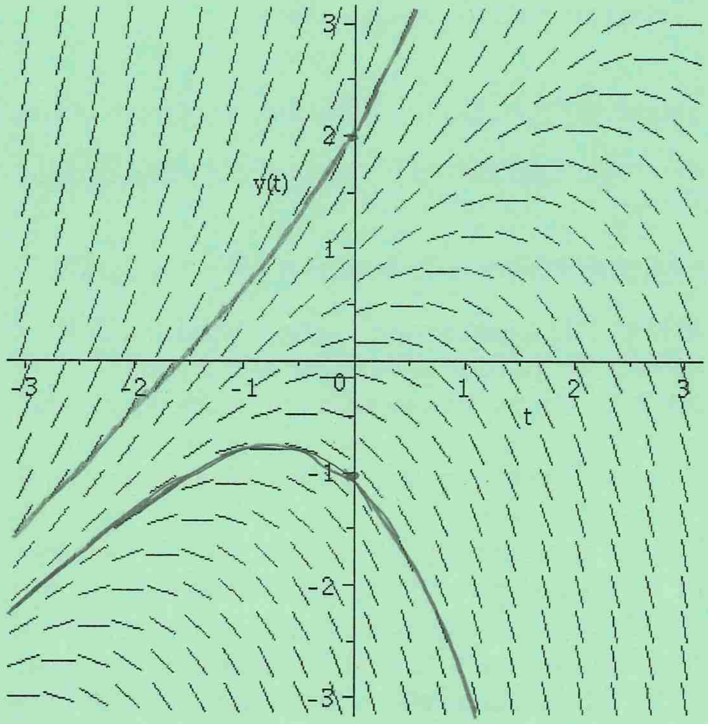
Differential Equation Test 1 Name ________________

Work each of the following problems analytically on the blank paper provided. Show all work in a neat, orderly manner. No credit will be given for answers without work involved in arriving at the answers.

1. Sketch an appropriate solution curve that passes through each of the indicated points. Label each sketch with the letter of the point.

4 A) $y(0) = -1$

4 B) $y(0) = 2$



2. A) Show that $y = \dfrac{-1}{x+C}$ is a solution to the DE $y' - y^2 = 0$.
6

4 B) Given the solution in part A, find a solution to the IVP consisting of the given DE and the initial condition $y(2) = 4$.

6 3. Find c_1 and c_2 so that $y(x) = c_1e^x + c_2e^{-x}$ is a solution which will satisfy $y(0) = 6$ and $y'(0) = -3$.

4. Consider the DE $\dfrac{dy}{dx} = y(y+2)^2(y-1)$:

7 A) Construct a phase portrait for the differential equation.

3 B) Classify each of the critical points as stable, unstable, or semi-stable.

2 C) For the initial condition $y(0) = 3$, what is the basic shape of the solution curve $y(t)$?

2A. (6) $y = \frac{-1}{x+c} = -(x+c)^{-1}$ $y' = (x+c)^{-2}$

$y' - y^2 = 0$

$(x+c)^{-2} - \left(-(x+c)^{-1}\right)^2 = 0$

$(x+c)^{-2} - (x+c)^{-2} = 0$ ✓

B. (4) $y(2) = 4$

$4 = \frac{-1}{2+c}$

$8 + 4c = -1$ $4c = -9$ $c = -\frac{9}{4}$ $\boxed{y = \frac{-1}{x - \frac{9}{4}}}$

3. (6) $y = c_1 e^x + c_2 e^{-x}$ $y' = c_1 e^x - c_2 e^{-x}$

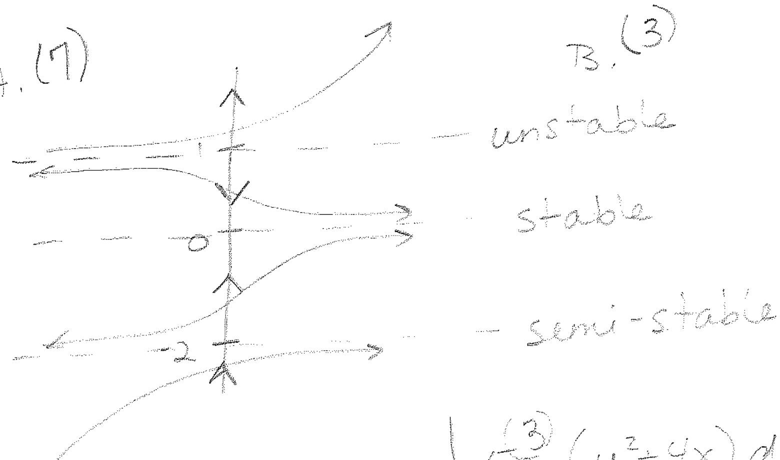
$y(0) = 6$: $6 = c_1 + c_2$ $y'(0) = -3$: $-3 = c_1 - c_2$

$6 = c_1 + c_2$
$-3 = c_1 - c_2$
$3 = 2c_1$
$\boxed{c_1 = 3/2}$

$6 = 3/2 + c_2$
$\boxed{c_2 = 9/2}$

4. $y = 0$
$y = -2$
$y = 1$

A. (7)



B. (3)

C. (2)

5. (9) $y^3\, dy = \left[e^{2x} + \cos(x)\right] dx$

$\frac{1}{4} y^4 = \frac{1}{2} e^{2x} + \sin(x) + c$

$y(0) = 2$ $4 = \frac{1}{2} + c \Rightarrow c = \frac{7}{2}$

$\frac{1}{4} y^4 = \frac{1}{2} e^{2x} + \sin(x) + \frac{7}{2}$

$y^4 = 2e^{2x} + 4\sin(x) + 14$

5. (3) $(y^2 + 4x)\,dx + (2xy + \sin y)\,dy = 0$

$M_y = 2y$ $N_x = 2y$ ✓ Exact

(8) $\frac{\partial f}{\partial x} = y^2 + 4x$

$f = xy^2 + 2x^2 + h(y)$

$\frac{\partial f}{\partial y} = 2xy + h'(y)$

$h'(y) = \sin y$

$h(y) = -\cos(y) + c_1$

$\boxed{xy^2 + 2x^2 - \cos(y) = c}$

6. $(3xy+y^2)\,dx+(x^2+xy)\,dy=0$

(7) $M_y=3x+2y \qquad N_x=2x+y$

$$\frac{M_y-N_x}{N}=\frac{\cancel{x+y}}{x\cancel{(x+y)}}=\frac{1}{x} \qquad I=e^{\int \frac{1}{x}dx}=x$$

$$\boxed{I(x)=x}$$

7. $x\frac{dy}{dx}-y=x^2\sin(3x)$ $\qquad I=e^{\int -\frac{1}{x}dx}=e^{-\ln(x)}=\frac{1}{x}$

(9)

$$\frac{dy}{dx}-\frac{1}{x}y=x\sin(3x)$$

$$\frac{1}{x}\frac{dy}{dx}-\frac{1}{x^2}y=\sin(3x)$$

$$\frac{d}{dx}\left[\frac{1}{x}y\right]=\sin(3x)$$

$$\frac{1}{x}y=-\frac{1}{3}\cos(3x)+c$$

$$\boxed{y=-\frac{1}{3}x\cos(3x)+cx}$$

8A. $N(0)=10$ $\qquad \frac{dN}{dt}=kN \qquad N=e^{kt+c}$

(7) $N(9)=5$ $\qquad \frac{1}{N}dN=k\,dt \qquad N=c_1e^{kt} \qquad 10=c_1e^0$

$\ln N=kt+c \qquad N=10e^{kt}$

$5=10e^{9k}$

$\frac{1}{2}=e^{9k} \Rightarrow k=\frac{\ln(1/2)}{9}$

$$N=10e^{\ln(1/2)/9\,t}$$

or

$$N=10e^{-\ln 2/9\,t}$$

(3)

B. $2=10e^{-\frac{\ln 2}{9}t} \qquad -\frac{9\ln(1/5)}{\ln 2}=t \qquad t=20.9$ years

9A. $T(0)=200^\circ$ $\qquad \frac{dT}{dt}=k(T-70)$

(4) $T_m=70^\circ$

$T(5)=125^\circ$

B. $\frac{1}{T-70}\,dT = k\,dt$

(5)

$\ln(T-70)=kt+c$

$T-70=e^{kt}\cdot e^{c}$

$200-70=e^{c}$

$130=e^{c}$

$T-70=130e^{kt}$

$125-70=130e^{k(5)}$

$\frac{55}{130}=e^{k(5)}$

$k=\frac{\ln(11/26)}{5}\approx -.172$

$T-70=130e^{\frac{\ln(11/26)}{5}t}$

10. (2) $\boxed{B.\ A(0)=20}$

$R_{in}=5\,\text{gal/min}\cdot 0\,\text{lb/gal}=0$

$R_{out}=\frac{A}{70+2t}\cdot 3\,\text{gal/min}=\frac{3A}{70+2t}$

(4) $\boxed{A.\ \frac{dA}{dt}=-\frac{3A}{70+2t}}$

$\frac{dA}{dt}+\frac{3}{70+2t}A=0$

$I=e^{\int\frac{3}{70+2t}dt}=e^{3/2\ln(70+2t)}=(70+2t)^{3/2}$

$\frac{d}{dt}\left[(70+2t)^{3/2}A\right]=0$

$(70+2t)^{3/2}A=C$

$A=C(70+2t)^{-3/2}$

$20=C(70)^{-3/2}$

$C=20\cdot 70^{3/2}$

$C=11713.24$

$A=11713.24(70+2t)^{-3/2}$

(2) $\boxed{C.\ 15\text{ minutes}}$

(2) $\boxed{D.\ \text{Zero}}$